The Commuting Graphs for Metabelian Groups of Order up to 24 with Their Properties

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Abstract
Let G be a finite group and let $Z(G)$ be the center of G. The commuting graph, denoted by $\Gamma(G)$, is a graph whose vertices are non-central elements of G i.e $|V\,\Gamma(G)| = |G| - |Z(G)|$ in which two vertices are adjacent if they commute. In this paper we present the commuting graph of all non-abelian metabelian groups of order less than 24, with its properties which includes the independent number, the chromatic number, the clique number and the dominating number.

Key Words: commuting graph, metabelian group, clique number, chromatic number.

1. Introduction

This section provides some background related to the metabelian groups and graph theory, suppose G is a finite non-abelian group. We begin with metabelian group. In 2010, Abdul Rahman has found all metabelian groups of order at most 24, there are 59 groups of order less than 24 with their presentations including abelian and non-abelian groups. A metabelian group is a group whose commutator subgroups are abelian. Equivalently, a group G is metabelian if and only if there exists an abelian normal subgroup A such that the quotient group G/A is abelian (Rose, 1994). By a metabelian group is meant a nilpotent group of nilpotency class two (Kurosh, 2014). A metabelian group is also a solvable group of derived length two (Neumann, 2012). In the following, we state fundamental concepts related to graph theory which are needed in this paper.

Graph theory is the study of vertices and edges. More precisely, it involves the way in which sets of points can be connected by edges. The concept in graph theory is widely used among many fields and one of these uses is in group theory. The graph Γ is connected if it has precisely one component. However, a graph is called a complete graph if each pair of distinct vertices are adjacent, and it is denoted by K_n, where n is the number of adjacent vertices (Godsil and Royle, 2001). The graph is called empty if there is no adjacent between its vertices. In addition, a graph is called null if it has no vertices and in this paper we denote K_0 the null graph (Bondy and Murty, 1982) and (Kurosh, 2014). Furthermore, a non-empty set S of $V(\Gamma)$ is called an independent set of Γ if there is no adjacent between two elements of S in Γ.

2. Preliminaries

In this section, some resent works that are needed in this paper are includes. This section is divided into two parts. The first part provides some definitions of properties of graphs. The second part, states the list of metabelian groups of order up to 24.

2.1 Properties of Graphs

We restate some graph properties that are needed in this paper.

Definition 2.1.1: Independent Number (Erfanian and Tolue, 2012)
Anon empty set S of $V(\Gamma)$ is called an independent set of Γ if there is no adjacent between two elements of S in Γ, while the independent number is the number of vertices in the maximum independent set and is denoted by $\alpha(\Gamma)$.

Definition 2.1.2: Chromatic Number (Erfanian and Tolue, 2012)
Chromatic number is the smallest number of colours needed to colour the vertices of Γ so that there will be no two adjacent vertices share the same colour and denoted by $\chi(\Gamma)$.

Definition 2.1.3: Dominating Number (Erfanian and Tolue, 2012)
The dominating set $X \leq V(\Gamma)$ is the set such that for each $v \notin X$, there exists x in $v \notin X$ such that v adjacent to x. The minimum size of X is called the dominating number and it is denoted by $\gamma(\Gamma)$.

Definition 2.1.4: Clique Number (Erfanian and Tolue, 2012)
Clique is a complete subgraph in Γ. The clique number is the size of the largest clique Γ and it is denoted by $\omega(\Gamma)$.

2.2 List of Metabelian Groups of Order up to 24

In 2010, Abdul Rahman (Abdul Rahman, 2010) classified all non-abelian metabelian groups of order up to 24 into 25 groups. The groups are stated as follows:

(1) $D_3 \cong \langle a,b : a^3 = b^2 = 1, bab = a^{-1} \rangle$

(2) $D_4 \cong \langle a,b : a^4 = b^2 = 1, bab = a^{-1} \rangle$

(3) $Q_8 \cong \langle a,b : a^4 = 1, a^2 = b^2, b^{-1}ab = a^{-1} \rangle$

(4) $D_5 \cong \langle a,b : a^5 = b^2 = 1, bab = a^{-1} \rangle$

(5) $\mathrm{Z}_3 \tilde{a}\ \mathrm{Z}_4 \cong \langle a,b : a^4 = b^3 = 1, bab = a \rangle$

(6) $A_4 \cong \langle a,b,c : a^2 = b^2 = c^3 = 1, ba = ab, ca = abc, cb = ac \rangle$

(7) $D_6 \cong \langle a,b : a^6 = b^2 = 1, bab = a^{-1} \rangle$

(8) $D_7 \cong \langle a,b,: a^7 = b^2 = 1, bab = a^{-1} \rangle$

(9) $D_8 \cong \langle a,b,: a^8 = b^2 = 1, bab = a^{-1} \rangle$

(10) Quasi-dihedral group, $G \cong \langle a,b : a^8 = b^2 = 1, bab = a^3 \rangle$

(11) $Q_{16} \cong \langle a,b : a^8 = 1, a^4 = b^2 = 1, aba = b \rangle$

(12) $D_4 \times \mathrm{Z}_2 \cong \langle a,b,c : a^4 = b^2 = c^2 = 1, ac = ca, bc = cb, bab = a^{-1} \rangle$

(13) $\mathrm{Z}_2 \times Q_4 \cong \langle a,b,c : a^4 = b^4 = c^2 = 1, ba = a^3b, ac = ca, ba = cb \rangle$

(14) $Modular-16 \cong \langle a,b : a^8 = b^2 = 1, ab = ba^5 \rangle$

(15) $B \cong \langle a,b : a^4 = b^4 = 1, ab = ba^3 \rangle$

(16) $K \cong \langle a,b,c : a^4 = b^2 = c^2 = 1, bab = a, ac = ca \rangle$

(17) $G_{4,4} \cong \langle a,b : a^4 = b^4 = 1, abab = 1, ab^3 = ba^3 \rangle$

(18) $D_9 \cong \langle a,b : a^9 = b^2 = 1, bab = a^{-1} \rangle$

(19) $S_3 \times \mathrm{Z}_3 \cong \langle a,b,c : a^3 = b^2 = c^3 = 1, bab = a^{-1}, ac = ca, bc = cb \rangle$

(20) $(\mathrm{Z}_3 \times \mathrm{Z}_3) \tilde{a}\ \mathrm{Z}_2 \cong \langle a,b,c : a^2 = b^3 = c^3 = 1, ba = cb, bab = a, cac = a \rangle$

(21) $D_{10} \cong \langle a,b : a^{10} = b^2 = 1, bab = a^{-1} \rangle$

(22) $F_{r20} \cong \mathrm{Z}_5 \tilde{a}\ \mathrm{Z}_4 \cong \langle a,b : a^4 = b^5 = 1, ba = ab^2 \rangle$

(23) $\mathrm{Z}_5 \tilde{a}\ \mathrm{Z}_4 \cong \langle a,b : a^4 = b^5 = 1, bab = a \rangle$

(24) $F_{r21} \cong \mathrm{Z}_7 \tilde{a}\ \mathrm{Z}_3 \cong \langle a,b : a^3 = b^7 = 1, ba = ab^2 \rangle$

(25) $D_{11} \cong \langle a,b : a^{11} = b^2 = 1, bab = a^{-1} \rangle$

3. Main Results

Throughout this section, $n \geq 3$ is an integer and there is 25 non-abelian metabelian groups of order less than 24. In this section, we commute the commuting graph for non-abelian metabelian groups of order less than 24. The result of the commuting graph on dihedral groups is investigated by (Chelvama et al., 2011).

Theorem 3.1

Let $G = Q_8 \cong \langle a,b : a^4 = 1, a^2 = b^2 =, b^{-1}ab = a^{-1} \rangle$ be the Quaternion group of order 8. Then, the commuting graph of G, $\Gamma(G) = K_2 \cup K_2 \cup K_2$.

Proof: The center of a group G, denoted by $Z(G)$, is defined as $Z(G) = \{x \in G : xy = yx, \textit{for all}\, y \in G\}$, then $Z(G) = \{e, a^2\}$. Since the order of G is 8 and the order of the center is 2, hence, the number of vertices of commuting graph Q_8 is 6, i.e $V(\Gamma(G)) = |G| - |Z(G) = |8 - 2 = 6$.

Table 3.1: The Cayley Table of Q_8

.	e	a	a^3	b	a^2b	a^2	ab	a^3b
e	e	a	a^3	b	a^2b	a^2	ab	a^3b
a	a	a^2	e	ab	a^3b	a^3	a^2b	b
a^3	a^3	e	a^2	a^3b	ab	a	b	a^2b
b	b	a^3b	ab	a^2	e	a^2b	a	a^3
a^2b	a^2b	ab	a^3b	e	a^2	b	a^3	a
a^2	a^2	a^3	a	a^2b	b	e	a^3b	ab
ab	ab	b	a^2b	a^3	a	a^3b	a^2	e
a^3b	a^3b	a^2b	b	a	a^3	ab	e	a^2

i. Let $x = a$ and y be any element in Q_8. Then the product of xy given as follows:

$(a)(a) = a^2$

$e = (a)(a^3) = (a^3)(a) = e$

$ab = (a)(b) \neq (b)(a) = a^3b$

$a^3b = (a)(a^2b) \neq (a^2b)(a) = ab$

$a^2b = (a)(ab) \neq (ab)(a) = b$

$b = (a)(a^3b) \neq (a^3b)(b) = a^2b$

Thus, a commutes with a^3 and itself.

ii. Let $x = b$ and y be any element in Q_8. then the product of xy given as follows:

$a^3b = (b)(a) \neq (a)(b) = ab$

$ab = (b)(a^3) \neq (a^3)(b) = a^3b$

$a^2 = (b)(b) = (b)(b) = a^2$

$e = (b)(a^2b) = (a^2b)(b) = e$

$a = (b)(ab) \neq (ab)(b) = a^3$

$a^3 = (b)(a^3b) \neq (a^3b)(b) = a$

Thus, b commutes with a^2b and itself.

iii. Let $x = b$ and y be any element in Q_8. then the product of xy given as follows:

$b = (ab)(a) \neq (a)(ab) = a^2b$

$a^2b = (ab)(a^3) \neq (a^3)(ab) = b$

$a^3 = (ab)(b) \neq (b)(ab) = a$

$a = (ab)(a^2b) \neq (a^2b)(ab) = a^3$

$a^2 = (ab)(ab) = (ab)(ab) = a^2$

$e = (ab)(a^3b) = (a^3b)(ab) = e$

Thus, ab commutes with a^3b and itself.

From the calculation above, a commutes with a^3, b commutes with a^2b and ab commutes with a^3b , Hence, the commuting graph of G is presented in figure 3.1.

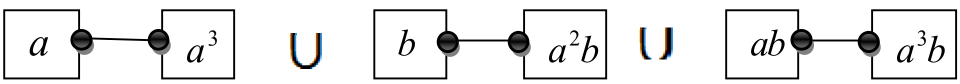

Figure 3.1: The commuting graph of Q_8, $\Gamma(G) = K_2 \cup K_2 \cup K_2$.

Theorem 3.2: Let G be a metabelian group of order 12, $\mathrm{Z}_3 \text{ ã } \mathrm{Z}_4 \cong \langle a,b : a^4 = b^3 = 1, bab = a \rangle$. Then, $\Gamma(\mathrm{Z}_3 \text{ ã } \mathrm{Z}_4) = K_4 \cup K_2 \cup K_2 \cup K_2$.

Proof: $G = \mathrm{Z}_3 \text{ ã } \mathrm{Z}_4 = \{e,a,b,ab,a^2,a^3,b^2,a^2b,a^3b,ab^2,a^2b^2,a^3b^2\}$, then $Z(G) = \{e,a^2\}$. Since the order of G is 12 and order of the center is 2, hence, the number of vertices of commuting graph $\mathrm{Z}_3 \text{ ã } \mathrm{Z}_4$ is 10, that is, $V(\Gamma(G)) = |G| - |Z(G)| = 12 - 1 = 10$. Since, a commutes with a^3, ab^2 commutes with a^3b^2, b commutes with a^2b, b^2, a^2b^2 and ab commutes with a^3b. Hence, the commuting graph of G is $\Gamma(\mathrm{Z}_3 \text{ ã } \mathrm{Z}_4) = K_4 \cup K_2 \cup K_2 \cup K_2$.

Theorem 3.3: Let G be a metabelian group of order 12, $A_4 \cong \{a,b : a^2 = b^2 = c^3 = 1, ba = ab, ca = abc, cb = ac\}$. Then, $\Gamma(A_4) = K_3 \cup K_2 \cup K_2 \cup K_2 \cup K_2$.

Proof: $G = A_4 = \{e,a,b,c,ab,ac,ba,ca,c^2b,ac^2,bc^2,c^2\}$, then $Z(G) = \{e\}$. Since the order of G is 12 and the order of the center is 1, hence, the number of vertices of commuting graph A_4 is 11, that is, $V(\Gamma(G)) = |G| - |Z(G)| = 12 - 1 = 11$, and since c commute with c^2, b commutes with ab and a, ac commutes with bc^2 and ca commutes with bc, ac^2. Hence, the commuting graph of G is $\Gamma(A_4) = K_3 \cup K_2 \cup K_2 \cup K_2 \cup K_2$.

Theorem 3.4: Let G be a metabelian group Quasi-dihedral of order 16, $G \cong \langle a,b : a^8 = b^2 = 1, bab = a^3 \rangle$. Then, $\Gamma(G) = K_6 \cup K_2 \cup K_2 \cup K_2 \cup K_2$.

Proof: $G = \{e,a,b,a^2,a^3,a^4,a^5,a^6,a^7,ab,a^2b,aba,ba,ba^7,aba^7,a^7b\}$, then $Z(G) = \{e,a^4\}$. Since the order of G is 16 and the order of the center is 2, hence, the number of vertices of commuting graph G is 14, that is $V(\Gamma(G)) = |G| - |Z(G)| = 16 - 2 = 14$, and b commutes with aba, ab commutes with ba^7, a^2b commutes with aba^7, ba commutes with a^7b and a commutes with a^2,a^3,a^4,a^5,a^6,a^7. Hence, the commuting graph of G is $\Gamma(G) = K_6 \cup K_2 \cup K_2 \cup K_2 \cup K_2$.

Remark 3.1: The commuting graph of metabelian group of order 16, namely Q_{16}, is the same as commuting graph in Theorem 3.4.

Theorem 3.5: Let G be a metabelian group of order 16, $D_4 \times Z_2 = \langle a,b,c : a^4 = b^2 = c^2 = 1, ac = ca, bc = cb, bab = a^{-1} \rangle$. Then, $\Gamma(D_4 \times Z_2) = K_4 \cup K_4 \cup K_4$.

Proof: $G = D_4 \times Z_2 = \{e,a,a^2,a^3,b,c,ab,ac,bc,abc,a^2b,a^2c,a^3b,a^3c,a^2bc,a^3bc\}$, then $Z(G) = \{e,a^2,c,a^2c\}$. Since the order of G is 16 and the order of the center is 4, hence, the number of vertices of commuting graph G is 12, that is $V(\Gamma(G)) = |G| - |Z(G)| = 16 - 4 = 12$, and since a commutes with ac, a^3, a^3c, b commutes with bc, a^2b, a^2bc and ab commutes with abc, a^3b, a^3bc. Hence, the commuting graph of G is $\Gamma(D_4 \times Z_2) = K_4 \cup K_4 \cup K_4$.

Remark 3.2: The metabelian groups $D_4 \times Z_2$, $Z_2 \times Q_8$, *Modular*-16, B, K, and $G_{4,4}$ of order 16, have the same commuting set, therefore they have the same commuting graphs similar to the graphs in Theorem 3.5.

Theorem 3.6: Let G be a metabelian group of order 18, $S_3 \times Z_3 \cong \langle a,b,c : a^3 = b^2 = c^3 = 1, b = a^{-1}, ac = ca, bc = cb \rangle$. Then, $\Gamma(S_3 \times Z_3) = K_3 \cup K_3 \cup K_3 \cup K_6$.

Proof:
$G = S_3 \times Z_3 = \{e, a, b, bc^2, a^2b, c, c^2, ac^2, a^2, bc, a^2bc^2, ab, ac, a^2c^2, a^2bc, abc^2, a^2c, abc\}$, then $Z(G) = \{e, c, c^2\}$. Since the order of G is 18 and the order of the center is 3, hence, the number of vertices of commuting graph G is 15, that is, $V(\Gamma(G) = |G| - |Z(G)| = 18 - 3 = 15$, and since b commutes with bc^2, bc, a commutes with $ac^2, a^2, ac, a^2c^2, a^2c, a^2b$ commutes with a^2bc^2, a^2bc, and ab commutes with abc^2, abc. Hence, the commuting graph of G is $\Gamma(S_3 \times Z_3) = K_3 \cup K_3 \cup K_3 \cup K_6$.

Theorem 3.7: Let G be a metabelian group of order 18, $(Z_3 \text{ ã } Z_3)\text{ã } Z_2 \cong \langle a,b,c : a^2 = b^3 = c^3 = 1, ba = cb, bab = a, cac = a \rangle$. Then,
$\Gamma(Z_3 \text{ ã } Z_3)\text{ã } Z_2 =$
$K_8 \cup \{a\} \cup \{ab\} \cup \{ac\} \cup \{ab^2\} \cup \{abc\} \cup \{ac^2\} \cup \{ab^2c\} \cup \{abc^2\} \cup \{ab^2c^2\}$.

Proof: $G = (Z_3 \times Z_3)\text{ã } Z_2 = \{e, a, b, c, ab, ac, b^2, bc, c^2, ab^2, abc, ac^2, b^2c, bc^2, ab^2c, abc^2, abc^2, b^2c^2, ab^2c^2\}$, then $Z(G) = \{e\}$. Since the order of G is 18 and the order of the center is 1, hence, the number of vertices of commuting graph G is 17, that is, $V(\Gamma(G)) = |G| - |Z(G)|$=18-1=17, and since b commutes with $c, b^2, bc, c^2, b^2c, bc^2, b^2c^2$, and a, ab, ac, $ab^2, abc, ac^2, ab^2c, ab^2c^2$ are commute with itselfs. Hence, the commuting graph of G is $\Gamma(Z_3 \text{ ã } Z_3)\text{ã } Z_2 = K_8 \cup \{a\} \cup \{ab\} \cup \{ac\} \cup \{ab^2\} \cup \{abc\} \cup \{ac^2\} \cup \{ab^2c\} \cup \{abc^2\} \cup \{ab^2c^2\}$

Remark 3.3: The commuting graph of metabelian group of order 20, namely $Z_5 \text{ ã } Z_4$, is the same as commuting graph in Theorem 3.7.

Theorem 3.8: Let G be a metabelian group of order 20, $F_{r20} \cong Z_5 \text{ ã } Z_4 \cong \langle a,b : a^4 = b^5 = 1, ba = ab^2 \rangle$. Then, $\Gamma(F_{r20} \cong Z_5 \text{ ã } Z_4) = K_3 \cup K_3 \cup K_3 \cup K_3 \cup K_3 \cup K_4$.

Proof: $F_{r20} \cong Z_5 \text{ ã } Z_4 = \{e, a, b, ab, ba, aba, a^2, a^3, a^3b, a^2b, b^2, b^4a^3, b^3, ba^3, ab^4a, b^4, b^4a, ab^4, a^3b^4, a^2b^4\}$, then $Z(G) = \{e\}$. Since the order of G is 20 and the order of the center is 1, hence, the number of vertices of commuting graph G is 19, that is, $V(\Gamma(G)) = |G| - |Z(G)| = 20 - 1 = 19$, and since a commutes with a^2, a^3 and b commutes with b^2, b^3, b^4, a^3b commutes with b^4a, a^2b^4, and a^2b commutes with ba, a^3b^4. Hence, the commuting graph of G is $\Gamma(F_{r20} \cong Z_5 \text{ ã } Z_4) = K_3 \cup K_3 \cup K_3 \cup K_3 \cup K_3 \cup K_4$.

Theorem 3.9: Let G be a metabelian group of order 21, $F_{r21} \cong Z_7 \text{ ã } Z_3 \cong \{a, b : a^3 = b^7 = 1, ba = ab^2\rangle$. Then, $\Gamma(Z_7 \text{ ã } Z_3) = K_6 \cup K_2 \cup K_2 \cup K_2 \cup K_2 \cup K_2 \cup K_2 \cup K_2$.

Proof: $Z_7 \text{ ã } Z_3 = \{e, a, b, a^2, a^2b, b^2, ab, aba, ab^6a^2, ba, b^6a^2, aba^2, a^2b^6a^2, ba^2, b^4, a^2ba^2, b^6, ab^6a, b^6a, a^2b^6, ab^6\}$, then $Z(G) = \{e\}$. Since the order of G is 21 and the order of the center is 1, hence, the number of vertices of commuting graph of G is 19, that is, $V(\Gamma(G)) = |G| - |Z(G)| = 21 - 1 = 20$, and since a commutes with a^2, b commutes with $b^2, ab^6a^2, aba^2, b^4, b^6$. a^2b commutes with b^6a, ab commutes with b^6a^2, aba commutes with $a^2b^6a^2$, ba commutes with a^2b^6, ba^2 commutes with ab^6, a^2ba^2 commutes with ab^6a. Hence, the commuting graph of G is $\Gamma(Z_7 \text{ ã } Z_3) = K_6 \cup K_2 \cup K_2 \cup K_2 \cup K_2 \cup K_2 \cup K_2 \cup K_2$.

Proposition 3.1: Let $Q_8 \cong \langle a, b : a^4 = 1, a^2 = b^2, b^{-1}ab = a^{-1}\rangle$ be a non-abelian metabelian group of order 8. The clique number of the commuting graph of G is $\omega(\Gamma_{Q_8}^{comm}) = 2$.

Proof: Since the largest subgraph of Q_8 is K_2, therefore, the clique number, $\omega(\Gamma_{Q_8}^{comm}) = 2$.

Table 3.2: The Clique Number of Non-abelian Metabelian Groups of order up to 24

Non-Abelian Metabelian Groups	Clique No.	Non-Abelian Metabelian Groups	Clique No.
S_3	2	$Mudular-16$	4
D_4	2	B	4
D_5	4	K	4
A_4	3	$G_{4,4}$	4
Z_3 ã Z_4	4	D_9	8
D_6	4	$S_3 \times Z_3$	6
D_7	6	$(Z_3 \times Z_3)$ã Z_2	8
D_8	6	D_{10}	8
$Quasi-Dihedral\ Group$	6	$F_{r20} \cong Z_5$ã Z_4	4
$D_4 \times Z_2$	4	Z_5ã Z_4	8
$Z_2 \times Q_8$	6	Z_7ã Z_3	6
Q_{16}	6	D_{11}	10

Proposition 3.2: Let $Q_8 \cong \langle a,b : a^4 = 1, a^2 = b^2, b^{-1}ab = a^{-1} \rangle$ be a non-abelian metabelian group of order 8. The chromatic number of the commuting graph of G is $\chi\left(\Gamma_{Q_8}^{comm}\right) = 2$.

Proof: The chromatic number of the commuting graph of Q_8, $\chi\left(\Gamma_{Q_8}^{comm}\right) = 2$ is two since the vertices are adjacent in component is K_2. Thus, they must have different colors of vertices.

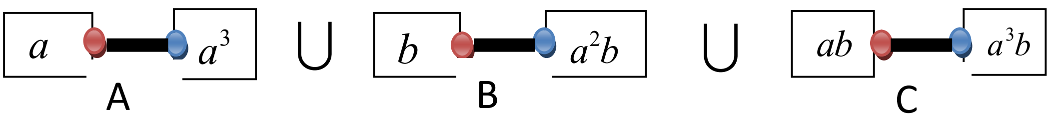

Figure 3.2: The Commuting Graph of Q_8, $\Gamma(G) = K_2 \cup K_2 \cup K_2$

Table 3.3: The Chromatic Number Of Non-Abelian Metabelian Group

Non-Abelian Metabelian Groups	Chromatic No.	Non-Abelian Metabelian Groups	Chromatic No.
S_3	2	$Mudular-16$	4
D_4	2	B	4
D_5	4	K	4
A_4	3	$G_{4,4}$	4
Z_3 ã Z_4	4	D_9	8
D_6	4	$S_3 \times Z_3$	6
D_7	6	$(Z_3 \times Z_3)$ã Z_2	8
D_8	6	D_{10}	8
$Quasi-Dihedral\ Group$	6	$F_{r20} \cong Z_5$ ã Z_4	4
$D_4 \times Z_2$	4	Z_5 ã Z_4	8
$Z_2 \times Q_8$	4	Z_7 ã Z_3	6
Q_{16}	6	D_{11}	10

Proposition 3.3: Let $Q_8 \cong \langle a,b : a^4 = 1, a^2 = b^2, b^{-1}ab = a^{-1} \rangle$ be a non-abelian metabelian group of order 8. The independent number of the commuting graph of G is $\alpha\left(\Gamma_{Q_8}^{comm}\right) = 3$.

Proof: From figure 3.2, the maximum independent set $= \{a,b,c\}$, where $a \in A, b \in B$ and $c \in C$ is 3. Since the independent number is the number of vertices in maximum independent set, thus $\alpha\left(\Gamma_{Q_8}^{comm}\right) = 3$.

Table 3.4: The Independent Number of Non-Abelian Metabelian Group

Non-Abelian Metabelian Groups	Independent No.	Non-Abelian Metabelian Groups	Independent No.
S_3	4	$Mudular-16$	3
D_4	3	B	3
D_5	6	K	3
A_4	5	$G_{4,4}$	3
Z_3 ã Z_4	4	D_9	10
D_6	4	$S_3 \times Z_3$	4
D_7	8	$(Z_3 \times Z_3)$ã Z_2	10
D_8	5	D_{10}	6
$Quasi-Dihedral\ Group$	5	$F_{r20} \cong Z_5$ ã Z_4	6
$D_4 \times Z_2$	3	Z_5 ã Z_4	6
$Z_2 \times Q_8$	3	Z_7 ã Z_3	8
Q_{16}	5	D_{11}	12

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پوختە

وادانێ G گروپێکی کۆتایی هاتووە و$Z(G)$ چەقی G. نەخشەی ئالۆگۆڕی, هێما دەکرێت بە $\Gamma(G)$ ئەو نەخشەیە کە خاڵەکانی بریتین لە دانەکانی G کە چەقی نین، واتە $|V\Gamma(G)| = |G| - |Z(G)|$، دوو لە دانەکان پێکەوە دەبەسترێن ئەگەر هاتوو ئالۆگۆڕبن. لەم توێژینەوەیەدا نەخشەی ئالۆگۆڕ دەخەینە روو بۆ گروپە نەئالۆگۆڕەکانی مێتائەبیلیەن گروپەکانی کە ژمارەی دانەکانیان لە ٢٤ کەمترە لەگەڵ سیفەتەکانیان، کە پێکدێت لە ژمارەی سەربەخۆ و ژمارەی کرۆماتیك و ژمارەی کلیك و ژمارەی دوومینەیتیك.

الخلاصة

لتكن G زمرة منتهية و ليكن $Z(G)$ هو مركز الزمرة G . البياني التباديلي يرمز له ب $\Gamma(G)$ هو بياني رؤوسه هي عناصر غير منتمية G لمركز الزمرة بمعنى $|V\Gamma(G)| = |G| - |Z(G)|$ بحيث كل رأسين يتجاوران إذا كانا تبديلين. في هذه الورقة، نقدم البياني التباديلي لزمرة الميتابيلين غير التبديلية للرتب الأقل من ٢٤ مع خصائص البياني متضمنة في العدد المستقل، العدد الأعظم للألوان، العدد الأعظم الجماعي و العدد المهيمن.